A CONTRIBUTION TO THE PROBLEM OF TURBULENT MASS TRANSFER AT HIGH VALUES OF THE SCHMIDT NUMBER AND TO THE HYDRODYNAMICS OF THE TURBULENT BOUNDARY LAYER

Václav KOLÁŘ, František VAŠÁK and Zdeněk BROŽ

Institute of Chemical Process Fundamentals,
Czechoslovak Academy of Sciences 165 02 Prague 6 — Suchdol

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It has been shown in the paper that under the turbulence ($Re > 10^4$) and at high values of the Schmidt number ($Sc > 10^3$), when the principal resistance to mass transfer is concentrated in the laminar layer immediately adhering to the interface, significant instabilities appear induced by the turbulent disturbances in the neighbouring transition layer, or by the discontinuities at the interface. Parameters have been determined characterizing this phenomenon on the basis of experimental data and their values have been compared with the data published in the literature as characteristics of the periodic viscous sublayer.

In the previous two communications[1,2] we have dealt with the theoretical description and experimental investigation of mass transfer under turbulence at high values of the Schmidt number ($Sc > 10^3$). An earlier published[3] model has been applied to obtain theoretical solution starting, as far as the hydrodynamics is concerned, from the classic concept, namely that in the immediate vicinity of the interface there is a laminar layer, followed by a transition layer, where the turbulent disturbances decay, and, finally the region of developed turbulence. At high values of the Schmidt number the principal resistance to mass transfer may be expected to be concentrated in the laminar layer. In addition, the entrance region, where the concentration field developes is so long making the determination of the mass transfer coefficient in laboratory conditions under the steady state, for which the earlier derived expression[3] holds impossible. Accordingly, the mass transfer has been solved in region of developing concentration profile[1], while making use of the earlier derived model[3]. For the average value of the mass transfer coefficient in a tube of circular cross section on a wall L long we thus obtained the following expression

$$K^+ = [(\lambda_p^+ \sqrt{(\pi)}/2 + \delta_I^+ Sc^{1/2})\, Sc^{1/2}]^{-1} - (1{\cdot}80866\, \delta_I^{+2}/L^+) \,. \tag{1}$$

$$. \sum_{i=1}^{\infty} A_i \varrho_i^{-4/3} [\exp(-\varrho_i^2 L^+/(\delta_I^{+3} Sc)) - 1] \,,$$

where δ_l^+ and λ_p^+ represent the thickness of the laminar and the transition region. In the cited paper[3] these parameters were evaluated from the mass transfer coefficients and were found to be

$$\delta_l^+ \approx 1 \quad (a) \quad \text{and} \quad \lambda_p^+ \approx 20 \quad (b). \tag{2a,b}$$

For the time scale of the turbulent disturbances in the transition layer the following expression was derived[3]

$$\tau_p = \lambda_p^2/\nu \tag{3}$$

which can be in turn rearranged to the following dimensionless form

$$\lambda_p^+ = u^* \sqrt{(\tau_p/\nu)} \equiv T^+ , \tag{4}$$

where T^+ designates the dimensionless time period introduced by Meek and coworkers[4] in the model of the periodic viscous sublayer. Meek's next paper[5] gives in its Fig. 1 measured values of the parameter T^+ obtained by various authors and various experimental techniques. Values of T^+ cover the range

$$14 < T^+ < 22 \tag{5}$$

which well agrees with the value

$$T^+ \approx 20 \tag{6}$$

following from Eqs (2b) and (4). Also the value δ_l^+ – according to Eq. (2a) agrees well with the idea of minimum thickness of the layer of the fluid remaining on the interface according to the model of the periodic viscous sublayer.

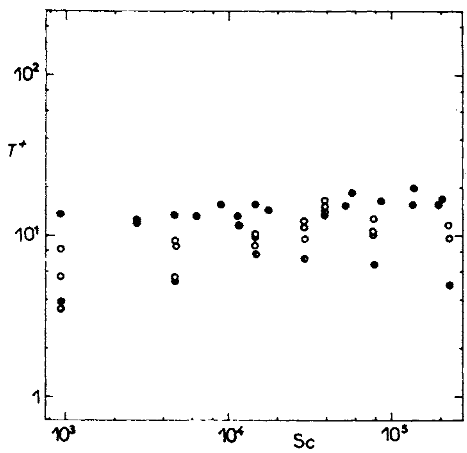

FIG. 1

The dimensionless period T^+ as a function of the Schmidt number computed from data of Vašák and coworkers[2] for rings 30 mm ○ and 10 mm ◐, and from data of Kishinivskii and coworkers[6] ◑, Grassmann and Tuma[8] ● for non-segmented experimental sections

Experimental verification of Eq. (*1*) for $Sc > 10^3$ revealed[2] that the theoretically predicted values of the mass transfer coefficient were lower than measured ones and that the difference grew with the increasing value of the Schmidt number. This discrepancy was ascribed to the conditions of the experiment. In this experiment the cylindrical interface was formed by the internal surface of segments of pressed benzoic acid of internal diameter 20 mm and 10 or 30 mm long. Thus, the imperfections of the size of individual segments gave rise to discontinuities of the interfacial surface at the point of contact of two adjacent segments, where mixing of the laminar film could well occur. If then only the length of a single cylindrical segment was substituted for L^+ into Eq. (*1*), the agreement between the computed and measured values of K^+ significantly improved.

The above reasoning, however, could not explain why the experimental values of the mass transfer coefficient, measured by Kishinevskii and coworkers[6] were also higher than the computed ones, although the experiment involved non-segmented cylindrical surface. It may thus be expected that there exists yet another source of periodic disturbances causing disruption of the concentration profile in the laminar layer. This source may well be periodic turbulent pulsations in the neighbouring transition layer, occuring with the period T^+, defined by Eq. (*4*). This equation can be modified by substituting for τ_p from the formal expression

$$\tau_p = L/u \qquad (7)$$

to obtain

$$(u^*L/\nu) = (u^*\lambda_p/\nu)^2 (u/u^*) \qquad (8a)$$

or

$$L^+ = T^{+2}u^+ , \qquad (8b)$$

where for T^+ should hold the value from Eq. (*5*).

Values of L^+ were computed from Eq. (*1*) with the aid of experimentally determined values of K^+. Formally the value of T^+ may be modelled[7] in terms of the mean velocity of the flow for which we may write

$$u_b^+ = \surd(2/f) \qquad (9)$$

and thus

$$T^+ = (L^+ \surd(f/2))^{1/2} . \qquad (10)$$

The values of L^+, computed from Eq. (*1*), and the values of T^+ computed from Eq. (*10*), are summarized in the Table I. Fig. 1 plots the function $T^+ = f(Sc)$. The Table as well as the figure give also the data found in the literature[6,8] for non-segmented experimental sections.

TABLE I

Values of parameters L^+ and T^+ computed from experimental data of Vašák and coworkers[2], Kishinevskii and coworkers[6], and Grassmann and Tuma[8] for various experimental conditions

Author	$L \cdot 10^3$	Sc	$Re \cdot 10^{-3}$	$K^+ \cdot 10^3$	L^+	T^+
Vašák and coworkers:		933	10·000	1·330	250	3·94
		4 683	10·000	0·380	439	5·26
		14 713	10·000	0·136	954	7·74
	10	29 969	10·000	0·089	842	7·25
		39 265	10·000	0·0472	3 245	14·22
		77 058	10·000	0·0465	651	6·57
		225 732	10·000	0·0292	406	5·03
			10·000	1·50	187	3·42
		933	20·00	1·00	544	5·58
			50·00	0·80	1 341	8·28
			9·297	0·362	498	5·6
		4 683	18·634	0·259	1 373	8·91
			50·00	0·244	1 724	9·39
			9·475	0·116	1 520	9·78
		14 713	20·00	0·111	1 776	10·09
			50·00	0·116	1 533	8·85
			10·00	0·0625	2 402	12·24
	30	29 696	20·00	0·0644	2 177	11·17
			45·185	0·0696	1 752	9·52
			10·00	0·0424	4 524	16·80
		39 265	20·00	0·0441	4 011	15·16
			38·45	0·0455	3 451	13·50
			10·00	0·0326	2 543	12·59
		77 058	20·00	0·0353	2 005	10·71
			28·00	0·0359	1 927	10·27
			10·00	0·0168	2 200	11·71
		225 732	16·48	0·0189	1 596	9·68
Kishinevskii and coworkers[6]		4 620	68·20	0·211	3 429	13·3
		6 260	68·40	0·166	3 475	13·11
		9 030	23·70	0·120	4 377	15·74
		11 400	49·50	0·117	2 616	11·61
		14 800	14·60	0·0845	4 154	15·79
		51 900	22·40	0·0363	4 159	15·38
	367	58 200	22·50	0·0295	6 054	18·54
		88 700	14·60	0·0242	4 703	16·18
		136 000	17·90	0·0189	4 189	15·65
		139 000	17·70	0·0159	6 824	20·00
		194 000	14·60	0·0148	4 214	15·95

TABLE I
(*Continued*)

Author	$L \,.\, 10^3$	Sc	$Re \,.\, 10^{-3}$	$K^+ \,.\, 10^3$	L^+	T^+
		202 000	14·60	0·0139	4 841	17·04
		930	85·40	0·684	3 947	13·77
		17 800	16·90	0·0782	3 599	14·56
Grassmann		11 190	10·00	0·1157	2 751	13·10
and Tuma[8]	346	11 190	20·00	0·1154	2 969	13·04
		2 769	10·00	0·3229	2 666	12·90
		2 769	20·00	0·3226	2 604	12·22

With respect to the physical fundamentals of the phenomenon the parameters L^+ and T^+ could be expected to depend on the Reynolds number. From the experimental data we had at our disposal, however, this dependence could not be determined. In Fig. 1 one can observe certain dependence of T^+ on the Schmidt number which, though, is not very significant. It is quite likely that this apparent dependence only reflects the fact that increasing value of the Schmidt number decreases the thickness of the diffusional boundary layer and hereby affects disruption of the concentration field. At sufficiently high value of this criterion this effect should be negligible. This was confirmed by the agreement of the theory with experiment at $Sc \sim 10^6$ in the previous work[2] and its Fig. 3.

From Fig. 1 it can be further seen that the assumption of the effect of discontinuities of the interfacial surface is very plausible for two reasons: The value of T^+ for both segmented sections is smaller than for the non-segmented ones and the value of T^+ grows with the length of the segment. From the table it is apparent that the mean value of T^+ obtained from experimental data on a non-segmented section[6,8] is

$$\overline{T^+} = 15{\cdot}44\,, \tag{11}$$

which well agrees with the value

$$T^+ = 15{\cdot}60\,, \tag{12}$$

which appears optimal for the experimental data from a number of papers[7].

It can be therefore concluded that the earlier derived relationship[1] describing the mass transfer mechanism at high values of the Schmidt number ($Sc > 10^3$) satisfies the experimentally found values[2] of the mass transfer coefficient. Nevertheless, the discontinuities at the interface, as well as the effect of turbulent disturbances in the transition layer, on the concentration profile in the laminar layer must be respected.

Further it can be concluded that the time scale of the disturbances, determined from experimental mass transfer data corresponds to the time scale in the model of periodic viscous sublayer and hereby contributes to the understanding of the existing mechanism.

LIST OF SYMBOLS

A_i defined in paper[1] by Eq. (*15*)
f friction factor
$\bar{K}$ mean value of the mass transfer coefficient averaged over the length L (m s^{-1})
$K^+ = \bar{K}/u^*$
L length scale of turbulent disturbance, in Eq. (*1*) and in Table the length of experimental section or length of a segment (m)
$L^+ = u^* L/\nu$
Re Reynolds number
Sc Schmidt number
u velocity component in direction of the flow (m s^{-1})
u_b mean velocity of the flow (m s^{-1})
$u^+ = u/u^*$
$u_b^+ = u_b/u^*$
$u^* = u_b \sqrt{(f/2)}$ (m s^{-1})
$T^+ = u^* \sqrt{(\tau_p/\nu)}$
δ_l thickness of laminar layer (m)
$\delta_l^+ = u^* \delta_l/\nu$
λ_p thickness of transition layer (m)
$\lambda_p^+ = u^* \lambda_p/\nu$
ν kinematic viscosity (m^2 s^{-1})
ϱ_i defined in paper[1] in Eq. (*14*)
τ_p time scale of turbulence in transition layer (s)

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